9. The velocity of planetary revolution at the Sun's surface : velocity of solar rotation, nearly : : Earth's orbital radius : Sun's radius.

$955{,}870 \div 4{,}421.7 = 216.173$; $216.173 \div 214.86 = 1.00613$.

10. The square of Jupiter's orbital radius : square of Earth's orbital radius, nearly : : g at Sun : g at Earth.

$27.292 \div 5.2028^2 = 1.00824$ (Compare No. 1).

ÆTHEREAL DENSITY AND POLARITY.

BY PLINY EARLE CHASE.

(Read before the American Philosophical Society, May 16th, 1872.)

If the conditions of equilibrium in a perfectly elastic gas have been disturbed by explosion, in the restoration of equilibrium, the particles will simultaneously rush towards each other, and towards the attractive centre m. If h is the extreme excursion consequent on the explosion, the centre of oscillation of each exploding particle being at $\frac{2h}{3}$, the centre of gyration of its return towards the centre of gaseous mass $\left(\frac{h}{2}\right)$ is at $\frac{5h}{9}$. The centre of gyration of the fall from $\frac{5h}{9}$ to the Earth, is at $\frac{5h}{27}$ above the Earth's surface, or at $r+\frac{5h}{27}$ from the Earth's centre.

If $h : r+\frac{5h}{27}$: : the orbital *vis viva* about a diameter $\frac{5h}{9}$: the *vis viva* which would be communicated by virtual fall through $\frac{5h}{9}$: : 1 : 4, we have $27r = 103h$; $\frac{5h}{9} = d' = 577.113$ miles ; $r = 91{,}345{,}800$ miles ; $\frac{r}{d} = 6.8666$; $\frac{68666}{78666}\sqrt{2gd'} = 6103$ feet per second. The approximation of the estimated velocity of hydrogen (6050, Clausius ; 6055, Joule) to this theoretical velocity, seems to indicate that the elasticity of hydrogen is nearly perfect. The inference is strengthened by the close approximation of my first estimates by flame analysis, to the mean of the best astronomical estimates of the Sun's distance.

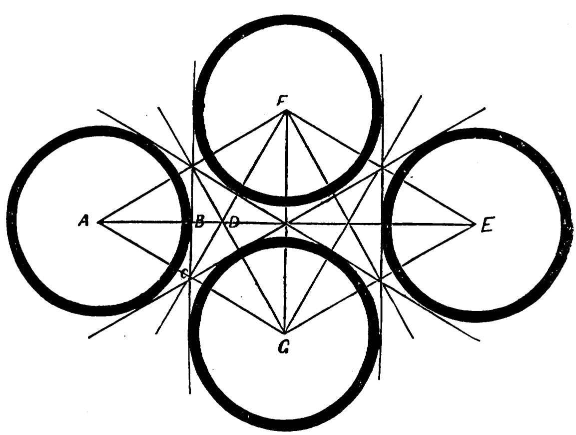
Let d'=density of luminiferous æther ; d''=density of hydrogen. Calling the velocity of sound in hydrogen 4163 feet, and the velocity of light 183,454 miles, if the elasticities are the same we have the proportion,

$d' : d''$: : $4163^2 : (183{,}454 \times 5280)^2$: : 1 : 54,130,000,000.

Upon the hypothesis that gravitation is an incidental result of æthereal

elasticity, Professor Norton has found* that "at the distance $80r$ the repulsion becomes very nearly the same for all the assumed values of $\frac{n}{m}$;" n and m being centres of origination for the interior and exterior wave systems of the æthereal atomic envelopes. Now if r=Sun's radius, and r'=Mercury's orbital radius, r' is nearly equivalent to $80r \times (\text{sec. } 30^\circ)^{\frac{1}{3}}$. The second factor may be deduced in the following manner:

If we suppose a collection of spherical atoms of equal magnitude, each of which has condensed around it an æthereal envelope, to be arranged in the most compact manner possible, the lines joining the centres of three contiguous particles will form an equilateral triangle. If the envelope is of such density as to allow the indefinite rectilinear transmission of waves between adjacent particles, one-half the distance between the centres of two adjacent atoms, will be a mean proportional between the radius of the atom and the distance between an atomic centre and an adjacent æthereal centre; the ratio of the atomic radius to the half-distance, and of the half-distance to the inter-central distance, being each equal to the ratio of radius to sec. 30°. In the accompanying figure,



$$AB : AC :: AC : AD :: \text{rad.} : \text{sec. } 30^\circ :: 1 : \sqrt{1.25}.$$

According to the hypothesis of Mossotti, that the æthereal particles are infinitesimal in proportion to the atoms, the sphericity of the atomic surface may be disregarded, and the virtual radius of the ætherally-enveloped atom may be supposed to vary as the mass. But in atoms of

* Amer. Jour. Sci., May, 1872, p. 340.

uniform density, $r \propto m^{\frac{1}{3}}$, therefore it need not surprise us if we find in different relations of cosmical masses, the three factors, sec. 30°, (sec. 30°)$^{\frac{1}{2}}$, (sec. 30°)$^{\frac{1}{3}}$.

1. Mercury's orbital radius being, as I have said, $r' = 80r \times (\text{sec. } 30°)^{\frac{1}{3}}$. Neptune's orbital radius is $80^2 r$, or $80r' \div (\text{sec. } 30°)^{\frac{1}{3}}$.

$$[80r \times (\text{sec. } 30°)^{\frac{1}{3}}] \div (.3870987 \times 214.86r) = 1.00126.$$
$$(30.03697 \times 214.86r) \div 6400r = 1.0084.$$

2. The distance of the exterior orbital limit of the asteroidal belt (Cybele) is nearly a mean proportional between the distances of Mercury and Neptune.

$$(.3870987 \times 30.03697)^{\frac{1}{2}} \div 3.4205 = 1.003115.$$

3. If the Moon's mass is $\frac{1}{83.032}$ of the Earth's mass, her distance is analogous to that of Mercury, being $80 \times (\text{sec. } 30°)^{\frac{1}{3}} \times$ the distance of the Earth's centre, from the centre of gravity of the terrestrial-lunar system. The mean of five recent estimates of the Moon's mass, given by Denison, is $\frac{1}{83.127}$.

$$83.127 \div 83.032 = 1.001156.$$

4. The year of Mercury : Neptune's year : : the velocity of a planet near the Sun's surface : the velocity of light, or nearly : : sec. 30° × the mass of the planetary system : the mass of the Sun.

$$(60126.72 \div 87.969258) \div (183,454 \div 265.5184) = 1.01088.$$
$$(60126.72 \times \text{sec. } 30°) \div (87.969258 \times 759.46) = 1.00617.$$

5. Venus may be regarded as an exterior satellite of the Earth, at a limit analogous to that of the solar system. For $80^2 \times$ Earth's radius nearly = Venus's distance at perigee.

$$80^2 \times 3962.818 = 25,362,050 ; \ 25,362,050 \div 25,268,000 = 1.0037.$$

6. The year of Uranus divided by (sec. 30°)$^{\frac{1}{3}}$, is equivalent to the month of a terrestrial satellite near the perigee distance of Venus (omitting considerations dependent upon solar acceleration and satellite-mass).

$$(25,268,000 \div 238,800)^{\frac{3}{2}} \times 27.32166 \times (\text{sec. } 30°)^{\frac{1}{3}} \div 30,686.821 = 1.006.$$

7. The mass of the intra-asteroidal planets : the mass of all the planets, nearly : : Sun's radius × (sec. 30°)$^{\frac{1}{3}}$: Earth's orbital radius.

$$(63.52 : 13167.12) \div [(\text{sec. } 30)^{\frac{1}{3}} : 214.86] = 1.00131.$$

My experiments in the years 1864 and 1865,* the most important of which were repeated before the Society, demonstrated that the simple mechanical action, of such elastic vibrations as are excited by the conjoint influence of solar radiation and terrestrial rotation, would not only produce polarity in a needle susceptible of similar vibrations, but it

* Proc. Amer. Philos. Soc. ix., 359; x., 151 sqq.

would also cause such daily and annual fluctuations as those of the magnetic needle. Professor Norton says* that "no conception has hitherto been formed of possible atomic movements capable of originating the electric forces, and producing even the simplest of the electrical phenomena." The dependence of the polarity of the compass upon electrical currents encourages the belief that all the "possible atomic movements capable of originating the electric forces," may be traceable to some arrangement of elastic æthereal particles like the one I have above suggested. The mutual attraction of the atoms A and E is only one-third as great as that of the atoms F and G, and under the influence of centrifugal force, the several lozenges AFEG would yield most readily in the direction EG.

THE SUN-SPOT CYCLE OF 11.07 YEARS.

BY PLINY EARLE CHASE.

(*Read before the American Philosophical Society, May 16th*, 1872.)

The most recent and careful discussion of the observations upon the amount and frequency of Sun-spots, by De La Rue, Stewart and Loewy, † assigns to the principal cycle a duration of 11.07 years. Kirkwood (ante, vol. xi., p. 100) had previously given nearly the same estimate (11.072 years). If the spots are attributable to disturbances produced by gravitating action, the major axis of the revolving disturbing force should be $[(11.07 \div 11.862)^{\frac{2}{3}}] = .955 \times$ Jupiter's, or 4.969 $\times$ the Earth's major axis. The mean radius vector of perturbation is, therefore, nearly equivalent to Jupiter's mean perihelion distance (.95184 ♃, or 4.952 ⊕) as well as to the mean distance of the centre of gyration of the planetary sytem.

$$\tfrac{2}{3} \times [(3\tfrac{1}{3} \times .3871 + 25 \times .7233 + 31.85 \times 1 + 3\tfrac{1}{3} \times 1.5237 + 9307 \times 5.2028 + 2847.4 \times 9.5389 + 416.7 \times 19.1826 + 532.5 \times 30.037) \div (3\tfrac{1}{3} + 25 + 31.85 + 3\tfrac{1}{3} + 9307 + 2847.4 + 416.7 + 532.5)] = 5.101 \; ; \; 5.101 \div 4.969 = 1.0265 \; ; \; 4.969 \div 4.952 = 1.0035.$$

The theoretical mean excursion between Jupiter's perihelion and his mean distance, corresponds very nearly with the above value of the planetary centre of gyration.

$$4.952 + \tfrac{5}{9} \times (5.2028 - 4.952) = 5.091 \; ; \; 5.101 \div 5.091 = 1.002.$$

If Jupiter's aphelion distance represents the aphelion distance of the aggregate of forces which produce the Sun-spots, the disturbance-perihelion is $(2 \times .955 - 1.048) = .862 \times$ ♃'s, or $4.485 \times$ ⊕'s radius vector. This corresponds very nearly with the linear centre of oscillation, of the mean

* Loc. cit., p. 330.

† Proc. Roy. Soc., Dec. 21, 1871; Phil. Mag., May, 1872.